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The direction of the National Observatory from the George-Town Observatory was observed to be 56° 44′ 40″ to the East of South. Hence, after allowing for the distance of the domes from the points of observation, the dome of the National Observatory is 47″·4 South and 6^{s}·2 East of the dome of George-Town College Observatory.

From Mr. Kluskey, City Surveyor of Washington, the following differences of latitude and departure between the dome of the Capitol and the National Observatory were obtained, viz.—

Capitol Dome 1906 feet S., and 12158·8 feet E. of the National Observatory.

Adopting the position of George-Town Observatory, given above, Mr. Curley finds the following latitudes and longitudes:—

	North Lat.			West Long.		
	°	′	″	h	m	s
George-Town College, Dome	38	54	26·0	5	8	18·20
National Observatory, Dome		53	38·58			12·00
Capitol — Dome		53	19·87			1·74
Old Naval Observatory, Lieut. Gilliss' Station	38	53	31·61	5	8	1·83

i.e. following Lieut. Gilliss' statement, that it was 1200 feet N.W. 5° from the Capitol Dome.

Mr. Curley adds that the substance of the above account is taken from a MS. written two or three years ago, which contains a description of the instruments and some observations. Circumstances have not permitted him to get the MS. printed.

On the Numerical Computation of the Values of Slowly-converging Series.

By Sir J. Lubbock, Bart. F.R.S. &c.

The numerical values of series which are incapable of direct summation, and which slowly converge, are sometimes required, as, for example, the quantities called b in the *Méc. Cél.*, which are so important in astronomy and so troublesome to compute.

Suppose the slowly converging series,

$$1 + x + x^2 + x^3 + \text{ \&c.}$$

of which the Napierian logarithm is

$$x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \text{ \&c.}$$

the series which expresses the Napierian logarithm evidently converges more rapidly than that which expresses the corresponding natural number; and as this is the case in other similar instances, it is desirable, especially with a view to tabulating numerical values, corresponding to different values of the variable, to have a ready mode of obtaining the numerical values of the coefficients in the series which expresses the logarithm of any series of this kind.

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Let the series in question be represented by

$$1 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3 + \&c.$$

and let the Napierian logarithm of this quantity be represented by

$$A_1 x + A_2 x^2 + A_3 x^3 + A_4 x^4 + \&c.$$

Differentiating,

$$A_1 + 2A_2 x + 3A_3 x^2 + 4A_4 x^3 + \&c.$$

$$= \frac{\alpha_1 + 2\alpha_2 x + 3\alpha_3 x^2 + 4\alpha_4 x^3 + \&c.}{1 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3 + \&c.}$$

Multiplying up and equating coefficients,

$$A_1 = \alpha_1 \qquad 2A_2 + A_1\alpha_1 = 2\alpha_2 \qquad 3A_3 + 2A_2\alpha_1 + A_1\alpha_2 = 3\alpha_3$$

$$A_2 = \alpha_2 - \frac{3}{2} A_1\alpha_1$$

$$A_3 = \alpha_3 - \frac{1}{3} A_1\alpha_2 - \frac{2}{3} A_2\alpha_1 \qquad A_4 = \alpha_4 - \frac{1}{4} A_1\alpha_3 - \frac{2}{4} A_2\alpha_2 - \frac{3}{4} A_3\alpha_1$$

$$A_5 = \alpha_5 - \frac{1}{5} A_1\alpha_4 - \frac{2}{5} A_2\alpha_3 - \frac{3}{5} A_3\alpha_2 - \frac{4}{5} A_4\alpha_1,$$

of which equations the law is simple and evident.

In order to illustrate these expressions by an example, suppose it were required to calculate the coefficients of the cosines of multiples of cos θ in the developement of

$$\{1 - A \cos\theta\}^{-\frac{3}{2}}$$

this series may be expressed by

$$\mathfrak{B}_0 + \mathfrak{B}_1 \cos\theta + \mathfrak{B}_2 \cos 2\theta + \mathfrak{B}_3 \cos 3\theta + \mathfrak{B}_4 \cos 4\theta + \&c.$$

If $a = \frac{1}{2}\left\{\sqrt{1+A} + \sqrt{1-A}\right\} \qquad a' = \frac{1}{2}\left\{\sqrt{1+A} - \sqrt{1-A}\right\}$

$$\mathfrak{B}_0 = \frac{1}{a_3}\left\{1 + \frac{3\cdot3}{2\cdot2}\frac{a'^2}{a^2} + \frac{3\cdot5\cdot3\cdot5}{2\cdot4\cdot2\cdot4}\frac{a'^4}{a^4} + \frac{3\cdot5\cdot7\cdot3\cdot5\cdot7}{2\cdot4\cdot6\cdot2\cdot4\cdot6}\frac{a'^6}{a^6} + \&c.\right\}$$

$$\mathfrak{B}_1 = \frac{2}{a^3}\left\{\frac{3a'}{2a}\right\}\left\{1 + \frac{5\cdot3}{4\cdot2}\frac{a'^2}{a^2} + \frac{5\cdot7\cdot3\cdot5}{4\cdot6\cdot2\cdot4}\frac{a'^4}{a^4} + \frac{5\cdot7\cdot9\cdot3\cdot5\cdot7}{4\cdot6\cdot8\cdot2\cdot4\cdot6}\frac{a'^6}{a^6} + \&c.\right\}$$

$$\mathfrak{B}_2 = \frac{2}{a^3}\left\{\frac{3\cdot5}{2\cdot4}\frac{a'^2}{a^2}\right\}\left\{1 + \frac{7\cdot3}{6\cdot2}\frac{a'^2}{a^2} + \frac{7\cdot9\cdot3\cdot5}{6\cdot8\cdot2\cdot4}\frac{a'^4}{a^4} + \frac{7\cdot9\cdot11\cdot3\cdot5\cdot7}{6\cdot8\cdot10\cdot2\cdot4\cdot6}\frac{a'^6}{a^6} + \&c.\right\}$$

the quantities $\mathfrak{B}$ are connected with the quantities b of the *Méc. Cél.* vol. i. p. 267, by the equations

$$\frac{1}{2} b^{(0)}_{\frac{3}{2}} = a^3\,\mathfrak{B}_4 \qquad b^{(1)}_{\frac{3}{2}} = a^3\,\mathfrak{B}_1 \qquad b^{(2)}_{\frac{3}{2}} = a^3\,\mathfrak{B}_2$$

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$$\text{If } \mathfrak{B}_\circ = \frac{1}{a_3}\mathbf{B}_\circ \quad \mathfrak{B}_1 = \frac{3\,a'}{a_4}\mathbf{B}_1 \quad \mathfrak{B}_2 = \frac{3\cdot 5\,a'^2}{4\,a_5}\quad \mathbf{B}_2, \text{ \&c. and}$$

If $\frac{a^{12}}{a^2} = x$, the expressions for $\mathbf{B}_0, \mathbf{B}_1, \mathbf{B}_2$ are of the form

$$1 + \alpha_1\, x + \alpha_2\, x^2 + \alpha_3\, x^3 + \text{\&c.};$$

and the following table, calculated by Mr. Farley, shows the values of A_1, A_2, A_3, &c., α_1, α_2, α_3, &c.; and it will be seen that the coefficients A *converge*, while, on the contrary, the coefficients α *diverge*.

	$\mathbf{B}_0$	$\mathbf{B}_{11}$	$\mathbf{B}_{22}$		$\mathbf{B}_0$	$\mathbf{B}_{11}$	$\mathbf{B}_{22}$
A_1	2·2500	1·5625	1·5326	α_1	2·2500	1·5625	1·5326
A_5	·4007	·3461	·3277	α_5	7·3280	3·2319	2·9896
A_{10}	·1992	·1838	·1731	α_{10}	13·6910	5·0339	4·4391
A_{15}	·1330	·1263	·1192	α_{15}	20·0560	6·7571	5·7684
A_{20}	·1010	·0964	·0919	α_{20}	26·4210	8·4490	7·0460

In computing an isolated case of $\mathfrak{B}$, it would not be worth while to procure the values of A; but in forming an extended table of the quantities $\mathfrak{B}$, they may be employed with advantage. The quantities A are also interesting, as affording the means of resolving any series into factors. See a recent memoir by M. Cauchy in the *Comptes Rendus.*

If the series

$$1 + \alpha_1\, x + \alpha_2\, x^2 + \alpha_3\, x^3 + \text{\&c.} = 1 + X,$$

$$\text{the logarithm} = X - \frac{X^2}{2} + \frac{X^3}{3} - \frac{X^4}{4} + \text{\&c.};$$

and, by expanding, the expression for the logarithm might be obtained in terms of α_1, α_2, α_3, &c.; but as the indices of the quantities increased, the law would not be easy to exhibit, and the equations would be altogether unmanageable for the computor.

Description of an Instrument for calculating the Correction of a Transit Observation for Collimation, Level, and Azimuth Errors. By Professor Challis.

"The collimation, level, and azimuth errors of a transit being respectively a, b, c, in seconds of space, the zenith distance of the observed object being z, and its north polar distance δ, the instrument I am about to describe performs the calculation of $(a + b \cos z + c \sin z) \times \frac{\operatorname{cosec} \delta}{15}$, that is, the reduction of the observed time of transit to the time of meridian transit. The manner of